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Estimation of Internal Migration by the National Growth Rate Method: An Alternative Approach

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This paper proposes an extension of the National Growth Rate Method (NGRM) for making flexible estimation of internal migration into a region or a city, by incorporating the migration pattern as a function of time. Several single- and two-parameter families of functions of time have been suggested for the purpose. The estimation procedure is outlined along with the data requirement. The flexible NGRM, like its precursor of improved NGRM, can provide separate estimates for both the net migrants and their natural increase, which can be subjected to socio-economic analysis for the motivations or implications of migration.

Keywords: Traditional NGRM, Flexible NGRM, Internal Migration Components, Incorporation of the Underlying Migration Pattern, Mean Value Theorem

JEL Classification: C02, J11

I. INTRODUCTION

It is well known that the National Growth Rate Method (NGRM) for calculation of internal migration "is a crude but fairly commonly used method" (Shryock *et al.* 1971) because of its minimal data requirement. In developing economies, data availability is an endemic problem, and for this reason, elaborate methods cannot be often employed. Naturally then one has to resort to the NGRM. For applications of this method, one may refer to Vamathevan (1961), DTRC (1961) and NPPP (1987). Various methods of measuring internal migration can be found in the manual UN (1970).

We have earlier identified in Rahman (1987) the source of crudeness in NGRM, and have developed the general procedure for removing the crudeness. This requires knowledge of the migration pattern that is not generally known. Thus, the migration pattern has to be assumed from prior evidence, although the actual migration pattern will be estimated alongside the estimation of the quantum of pure migrants. This is to be understood that the need to make

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assumptions about the migration pattern is not a fault of the present endeavours, because it is in the very nature of the problem that the quantum of migration will be dependent on the migration pattern.

Note that Rahman's (1987) assumption of uniform migration rate may be justified, via the Mean Value Theorem, as a stand-alone proximate method. And it can be considered as a first step to start with the problem, because this applies to each intercensal period of typically ten years on a piecemeal basis only, and the previous or the next intercensal period may well have a different uniform migration rate. That is, uniformity needs to hold only locally and approximately. Taking account of migration pattern with uniform migration rate assumption is definitely an improvement over the traditional NGRM that does not incorporate migration pattern, and it means better utilisation of available information that is otherwise ignored.

Rahman's second paper (1993) assumed that the migration rate increases with the population of the region/country, which can capture the main feature of the true migration pattern, i.e. its feature of increasing pattern is represented by Rahman's assumption. However, it is generally not to be expected that the migration pattern will exactly follow the population growth pattern whether it be of the region or of the country. This means that the employed form for migration pattern should be flexible enough to represent any possible pattern of migration. However, this raises several problems for estimation method and for data requirement. Thus, unlike Rahman's implicit representation of the migration pattern, we need to find extraneous variable that can do the same job. For this, the most obvious candidate is the time variable, which has the additional advantage that it can be interpreted to capture the composite effect of the socio-economic and demographic variables affecting migration pattern. However, some problems remain. For example, to keep within the requirement of data by the traditional NGRM, no more than one variable can be allowed to enter the form employed for the migration pattern expressed as a function of time. The difficulty is inherent in the nature of the problem rather than in our proposed solution. The two-parameter family of functions of time (t) will offer greater flexibility, but the cost is that for the additional parameter, we shall need additional data to make its estimation possible. Thus, one shall need data for three censuses instead of two censuses as for the one parameter family of curves.

II. DERIVATION OF ESTIMATING FORMULAE

The city is assumed to have experienced the same rate of natural increase as the whole nation, whence the method derives its name as the National Growth Rate Method, and the international migration is assumed to be zero or insignificant.[1] Thus, a city growth rate that is greater than the national growth rate is interpreted as net in-migration, and vice versa. Let P_o and P_T be the populations of a city at the beginning and the end of the intercensal period respectively, and let P'_o and P'_T be the corresponding national populations. Further, let t denote time, T denote the time elapsed during the intercensal period, m denote the number of persons entering the city per unit time, r denote the natural rate of population growth. In this section we shall obtain the estimating formulae of the migration rate (m), the pure migrants (M_o) and the natural increase of pure migrants (M_i), assuming that the migration rate m follows a time-dependent pattern.

We shall deal only with the general case of the two-parameter functions of time for the migration pattern, because it can give rise to the single-parameter functions as special case. The following possible forms for the migration rate, m=φ(t), shall be considered:

$$\varphi(t) = a + bt \tag{2.1}$$

$$\varphi(t) = a + b\log t \tag{2.2}$$

$$\varphi(t) = ae^{bt} \tag{2.3}$$

$$\varphi(t) = ab^{t} \tag{2.4}$$

where a and b are the two parameters to be determined to get the final result. The single-parameter family of functions can be obtained by dropping a in the first two equations and setting a equal to 1 in the last two equations. By obvious substitution in the two-parameter case formulae, the results for the single-parameter case can be obtained easily.

The general logic of the derivations can be articulated in the following way. The value of r, which will be treated often as given data, can be found from the following relation

$$P'_T = P'_o e^{rT} \tag{2.5}$$

or, $$r = (1/T)\log(P'_T/P'_o) \tag{2.6}$$

By our assumption of equal population increase rates, the size of the city population at the end of the intercensal period would have been

[1] One needs to add the qualification that the required conditions are just the ideals that need not be satisfied exactly in practice, or that they could be relaxed, via the Mean Value theorem, to hold only approximately. This could help avoid misunderstanding about the applicability of the NGRM when its assumptions do not hold perfectly.

$$P_T = P_o\, e^{rT} \tag{2.7}$$

Then the difference between the "observed city population" and the "expected city population in absence of internal migration" is

$$M = P_T - P_o e^{rT} \tag{2.8}$$

which using equation (2.5), can be put in the form

$$M = P_T - P_o (P'_T / P'_o) \tag{2.9}$$

which can be called the "contribution of migration." It can be divided into two components e.g. the "pure migrants" and the "natural increase of the migrants", written shortly as

$$M = M_o + M_i \tag{2.10}$$

It is apparent that our object is to obtain separate estimates of M_o and M_i. We shall first evaluate M_o, whence the latter M_i can be found easily. To do this, we note that the value of M is the integral,

$$M = \int_o^T m e^{r(T-t)} dt \tag{2.11}$$

and, the value of M_o is the integral

$$M_o = \int_o^T m dt \tag{2.12}$$

Now using the two fundamental relationships (2.11) and (2.12), the pure migration M_o can be evaluated for various time-dependent forms of migration rate, $m = \varphi(t)$. The natural increase of pure migrants M_i can be evaluated easily from the relation $(M_i = M - M_o)$ using equation (2.9), and will not be presented in subsequent presentation for the sake of space economy.

Case I: The Simple Linear Function

Here the form considered for migration rate is the simple linear function,

$$m = \phi(t) = a + bt \tag{2.13}$$

This function is the simplest of all possible forms, yet it is the obvious choice in the absence of prior information about the curvature of the migration pattern. For the sake of obtaining simpler final formulae, we shall generally choose the starting time as zero, except in Case II for reasons to be clarified in the next paragraph. From the general relationships (2.11) and (2.12), we have

$$M=\int_{o}^{T}(a+bt)e^{r(T-t)}dt \tag{2.14}$$

and, $M_o=\int_{o}^{T}(a+bt)dt$ (2.15)

On evaluation of the integral (2.14), we have

$$M=ar^{-1}(e^{rT}-1)+br^{-2}(e^{rT}-rT-1) \tag{2.16a}$$

Equating the right-hand-side of the equation (2.16a) with that of (2.8) and solving for b, we have

$$b=r(e^{rT}-rT-1)^{-1}\{r(P_T-P_o e^{rT})+a(1-e^{rT})\} \tag{2.16}$$

Evaluating the integral (2.15), we obtain the following expression

$$M_o=aT+\frac{1}{2}bT^2 \tag{2.17}$$

The next three functions from the previous list, which follow now consecutively, represent some curvature in the pattern of migration rate.

Case II: The Logarithmic Function

When the migration rate is considered to exhibit logarithmic pattern, the following form may be considered,

$$m=\varphi(t)=a+b\log t \tag{2.18}$$

Inserting (2.18) in the general relation (2.11), we have

$$M=\int_{L}^{T}(a+b\log t)e^{r(T-t)}dt \tag{2.19}$$

where the upper limit of time is T as usual, while the lower limit is not zero because 'log 0' does not exist or cannot be defined. The starting time *(L)* should be some positive number and preferably well above unity if one wants to avoid cumbersome values. Evaluating the integral in (2.19), we have

$$M=(a/r)(-1+e^{r(T-L)})+be^{rT}I(r,T,L) \tag{2.20}$$

where we define the function I(r,T,L) as

$$I(r,T,L)=\int_{L}^{T}(\log t)e^{-rt}dt \tag{2.20a}$$

Equating RHS of (2.20) with that of (2.8) and solving for b, we have

$$b=\frac{P_T-P_o e^{rT}}{I(r,T,L)e^{rT}}+\frac{a}{I(r,T,L)re^{rT}}\left\{1-e^{r(T-L)}\right\} \tag{2.21}$$

The expression for M_o is given by

$$M_o = \int_L^T (a + b\,logt)dt \qquad (2.21a)$$

which, on evaluation, reduces to

$$M_o = (a+b)(T-L) + b(T\,log\,T - L\,log\,L) \qquad (2.22)$$

Case III: The Exponential Function

The exponential pattern of the migration rate may be represented by the following function,

$$m = \varphi(t) = ae^{bt} \qquad (2.23)$$

Inserting (2.23) in the general relation (2.11), we have

$$M = \int_o^T ae^{bt} e^{r(T-t)} dt \qquad (2.24)$$

On evaluating the integral (2.24) and equating with (2.8), we have the relation

$$e^{rT} - e^{bT} = \frac{(b-r)}{a}(P_T - P_o e^{rT}) \qquad (2.25)$$

which does not yield any explicit solution for b, and must be solved by numerical methods. The expression for M_o can be obtained from the integral,

$$M_o = \int_o^T ae^{bt} dt \qquad (2.26)$$

which, on evaluation, gives

$$M_o = (a/b)(e^{bT} - 1) \qquad (2.27)$$

Case IV: The Power Function

Lastly, the power function pattern in the migration rate (*m*) may be represented by the following form,

$$m = \varphi(t) = ab^t \qquad (2.28)$$

Using (2.28) in the general relation (2.11), we have

$$M = \int_o^T ab^t e^{r(T-t)} dt \qquad (2.29)$$

On evaluating the integral (2.29), we have

$$M = \frac{a}{log\,b - r}(e^{bT} - e^{rT}) \qquad (2.30)$$

Equating RHS with that of (2.8), we have the relation

$$\frac{b^T - e^{rT}}{\log b - r} = \frac{P_T - P_o e^{rT}}{a} \tag{2.31}$$

which can be solved for b by numerical methods. Expression for M_o can be obtained in this case from the following integral

$$M_o = \int_o^T ab^t dt \tag{2.32}$$

which, on evaluation, results in the expression

$$M_o = \frac{a}{\log b}(b^T - 1) \tag{2.33}$$

It should be noted that in the above Cases I, III and IV, if the starting time is chosen to be non-zero for some special reason, one should accordingly define the lower limit of the relevant integrals that will need fresh evaluation, which we have not attempted here.

III. ESTIMATION PROCEDURE

Estimation of pure migrants (M_o) and migration rate (m) is entangled with the estimation of a and b appearing in the different time-dependent forms assumed for migration rate. For estimating a and b from the equations (2.16), (2.21), (2.25) and (2.31), two linearly independent equations should be generated from each of the aforementioned equations, which can be ensured by using city's and national population figures for three consecutive censuses. The steps to be followed for this purpose can be outlined in the following way.

Step 1: Estimate r from the national population figures from any pair of consecutive censuses for the three population censuses considered, using the relation (2.6).

Step 2: This step is needed only for the logarithmic specification of Case II. To evaluate $I(r,T,L)$ from equation (2.20a), appropriate numerical quadrature formula can be used after inserting the value of r obtained from Step 1. Notably, this is the most formidable task in the whole procedure required for Case II.

Step 3: Using the population totals (of the city concerned) of the first two consecutive censuses and the value of r obtained from the Step 1, an equation for estimating a and b should be formed, and the second equation will be similarly formed using the corresponding figures of last two censuses. Then the two equations are to be numerically solved for *a* and *b*.

Step 4: Using the above estimates of *a* and *b*, one obtains finally the estimates of M_o and *m* from the appropriate formulae of the previous section.

In the absence of dedicated software, the computations can be implemented conveniently by first developing an Excel spreadsheet for the purpose. This may save both time and energy, particularly if one has to perform repeated calculations or experiment with the figures.

Lastly, as in all models (or regressions) involving time as an explanatory variable, there is the inherent problem of arbitrariness in choosing the origin and scale of the time variable. Consequently, there will be certain effects of this feature on the value of the estimated parameters of the model, but these effects are of no consequence in practice, because it will create no problem in describing the observed data or in the model predictions, since these are not affected at all.

IV. CONCLUDING REMARKS

There are much more issues related to the flexible/improved NGRM than can be elucidated here. However, one can make the following remarks briefly.

The flexible/improved NGRM, like the traditional NGRM, can be applied to estimate migration into a region (or city), whenever simply the population data can be obtained for two points of time, and this requires no other demographic or socio-economic information. It is remarkable that the unknown pattern of migration can be taken into account through representing the migration pattern by a function of time.

The flexible NGRM, including its precursor of improved NGRM, can provide separate estimates for both the net migrants and their natural increase. On this count, the flexible/improved NGRM appears to excel the traditional NGRM, and, in fact, excel all other indirect methods for estimation of internal migration.

The NGRM is a mathematical method designed to make estimates of migration into a city (or region), but it is not meant to produce a causal model of migration. After one obtains estimates of migration components through the traditional NGRM or its present variants, they can be fed as input data to a causal model of migration for finding the underlying influencing factors, or they can be subjected to socio-economic analysis for the motivations or implications of migration.

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